Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1843. No. 37.

January 9.

SIR Wm. R. HAMILTON, LL.D., President, in the Chair.

Stewart Blacker, Esq., Thomas Cather, Esq., William V. Drury, M. D., William Gore, M. D., Thomas Hodder, Esq. R. N., Rev. John Homan, Henry Hutton, Esq., Robert Leslie Ogilby, Esq., the Hon. Frederick Ponsonby, and George Salmon, Esq., F.T.C.D., were elected members of the Academy.

Rev. H. Lloyd, V. P., read a paper " on the Determination of the Intensity of the Earth's Magnetic Force in absolute Measure."

The means of determining the intensity of the earth's magnetic force in absolute measure consist, it is well known, in observing the time of vibration of a freely-suspended horizontal magnet, under the influence of the earth alone, and then employing the same magnet to act upon another, which is also freely-suspended, and noting the effects of its action combined with that of the earth. From the former of these observations we deduce the *product* of the horizontal component of the earth's magnetic force into the moment of free magnetism of the first magnet,—from the latter, the *ratio* of the same quantities; and, the product and the ratio being thus known, the two factors are absolutely determined. The former part of this process involving no difficulty which may not be overcome by due care in observing, we shall confine our attention, in the present communication, to the latter.

Two methods have been proposed for this second observation, one by Poisson, and the other by Gauss. The method of Poisson consisted in observing the *time of vibration* of the second magnet, under the combined action of the first and of the earth, the acting magnet having its axis in the magnetic meridian passing through the centre of the suspended magnet. In the method of Gauss, which is now universally adopted, we observe the *position of equilibrium* of the second magnet, resulting from the action of the same forces. The acting magnet being placed transversely with respect to the suspended one, the latter is deflected from the meridian, and the amount of this deflection serves to determine the ratio of the deflecting force to the earth's force. The position chosen by Gauss for the deflecting magnet is that in which its axis is in the right line passing through the centre of the suspended magnet, and perpendicular to the magnetic meridian, in which case the tangent of the angle of deflection is equal to the ratio of the two forces. From this ratio it remains to deduce that of the magnetic moment of the deflecting bar to the earth's force.

The difficulty of this process arises from the form of the expression of the force of the deflecting bar. This force being expressed by a series descending according to the negative odd powers of the distance, with unknown coefficients, it is evident that observation must furnish as many equations of condition, corresponding to different distances, as there are terms of sensible magnitude in the series; and from these equations the unknown quantities are to be deduced by elimination. Now, the greater the number of unknown quantities thus eliminated, the greater will be the influence of the errors of observation on the final result; and if, on the other hand, the distance between the magnets be taken so great, that all the terms of the series after the first may be insensible, the angle of deflection becomes very small, and the errors in its observed value bear a large proportion to the whole.

It fortunately happens, that at moderate distances (distances not less than four times the length of the magnets) all the terms beyond the second may be neglected. The expression for the tangent of the angle of deflection is thus reduced to two terms, one of which contains the inverse cube of the distance, and the other the inverse fifth power; that is, if u denote the angle of deflection, and D the distance,

$$\tan u = \frac{Q}{D^3} + \frac{Q'}{D^5};$$

in which Q and Q' are unknown coefficients, the former of which is double of the ratio sought. Accordingly, the method recommended by Gauss consists in observing the angles of deflection, u and u', at two different distances, D and D', and inferring the coefficient Q by elimination between the two resulting equations of condition.

It is evident, however, that if the coefficient of the inverse fifth power of the distance be evanescent,—or, more generally, if the ratio of the two coefficients be known *à priori*,—the quantity sought may be obtained, without elimination, from the results of observation at one distance only. For if $Q' = hQ$, h being a known quantity, the preceding expression becomes

$$\tan u = \frac{Q}{D^3}\left(1 + \frac{h}{D^2}\right);$$

and accordingly the value of Q is obtained, from the result of observation at a single distance, by the formula

$$Q = \frac{D^3 \tan u}{1 + hD^{-2}}.$$

And, not only is the labour of observation thus diminished, but (which is of more importance) the accuracy of the result is increased. In order to show this, the author entered into an examination of the amount of the probable error in the two methods, from which it appeared that the probable error of Q, arising from an error in the observed deflection, will be less than in the usual method in the ratio of 1 to 5.563, even when the latter is employed in the manner most conducive to accuracy. In fact, the ratio of the probable error to the entire quantity is found to be expressed, in the two cases, by the formulæ

$$\frac{\Delta Q}{Q} = \frac{\Delta u}{u}, \quad \frac{\Delta Q}{Q} = \frac{\sqrt{q^{10}+1}}{q^2-1}\,\frac{\Delta u}{u}$$

where q denotes the ratio of the two distances; and the *least* value of the factor $\frac{\sqrt{q^{10}+1}}{q^2-1}$ is 5.563, and corresponds to the ratio $q = 1.32$.

In order to know the ratio h, it is necessary to determine the moment of the force exerted by the deflecting magnet upon the suspended magnet, extending the approximation to the terms involving the fifth power of the distance. The axis of the deflecting magnet being supposed to lie in the right line joining the centres of the two magnets, and the axis of the suspended magnet forming the angle ψ with that line, this moment is found to be

$$\frac{2MM'}{D^3}\sin\psi \left\{ 1 + \left(2\frac{M_3}{M} + 3(5\cos^2\psi - 1)\frac{M'_3}{M'} \right) \frac{1}{D^2} \right\};$$

in which M and M', M_3 and M'_3, denote certain integrals depending on the distribution of free magnetism, in the deflecting and suspended magnets, whose values are

$$M = \int_{-l}^{+l} mr\,dr, \quad M_3 = \int_{-l}^{+l} mr^3\,dr,$$

m being the quantity of free magnetism in any transverse section of the magnet, r its distance from the centre, and l half the length of the bar. The form of this result exhibits the advantage of the method of deflection recently proposed by Professor Lamont, in which $\psi = 90^\circ$, or the deflecting bar perpendicular to the suspended bar.

In the ordinary method, $\psi = 90^\circ - u$; and, the moment of the force exerted by the earth being $\mathrm{XM}' \sin u$, where X denotes the horizontal component of the earth's magnetic force, the equation of equilibrium is

$$\tan u = \frac{2\mathrm{M}}{\mathrm{X}} \left\{ \frac{1}{\mathrm{D}^3} + \left(2\frac{\mathrm{M}_3}{\mathrm{M}} - 3\frac{\mathrm{M}_3'}{\mathrm{M}'} + 15\sin^2 u \frac{\mathrm{M}_3'}{\mathrm{M}'} \right) \frac{1}{\mathrm{D}^5} \right\}.$$

The angle of deflection, u, being small, the term involving the square of its sine may be neglected, in comparison with the others; and the equation assumes the form already adverted to, namely,

$$\tan u = \frac{\mathrm{Q}}{\mathrm{D}^3} \left(1 + \frac{h}{\mathrm{D}^2} \right);$$

in which we have made, for abridgment,

$$\frac{2\mathrm{M}}{\mathrm{X}} = \mathrm{Q}, \quad 2\frac{\mathrm{M}_3}{\mathrm{M}} - 3\frac{\mathrm{M}'_3}{\mathrm{M}'} = h.$$

In order to apply this result, we must know, at least approximately, the law of magnetic distribution, or the function of r by which m is represented. Almost the only knowledge which we possess on this subject is that derived from the researches of Coulomb. From these researches M. Biot has inferred, that the quantity of free magnetism, in each point of a bar magnetized by the method of double touch, may be represented by the formula

$$m = \mathrm{A}(\mu^{l-r} - \mu^{l+r});$$

μ being a quantity independent of the length of the magnet, and A a function of μ and l. M. Biot has further shown, that when the length of the magnet is small, the relation between m and r is approximately expressed by the simple formula

$$m = m'\frac{r}{l};$$

the curve of intensities becoming, in that case, very nearly a right line passing through the centre of the magnet.

Employing then this approximate formula, we have

$$M = \tfrac{2}{3} m' l^2 ; \quad M_3 = \tfrac{2}{5} m' l^4 .$$

The ratio of these quantities is $\frac{M_3}{M} = \frac{3}{5} l^2$, a value independent of m'; and substituting in the expression of h above given, and designating the half lengths of the deflecting and of the suspended magnets by l and l', respectively,

$$h = \tfrac{3}{5} (2 l^2 - 3 l'^2) ;$$

an expression whose value may be exactly known, independently of experiment. This value vanishes, when $l^2 = \frac{3}{2} l'^2$, or

$$l = 1{\cdot}224\, l' ;$$

and in this case, therefore, the quantity sought is given by the simple formula

$$Q = D^3 \tan u .$$

The author concluded his paper with an account of a series of deflection experiments, instituted for the purpose of confirming these results. The magnets employed were cylindrical, their lengths being 3 inches and $3\frac{2}{3}$ inches, and their diameter 3–10ths of an inch. The observations were made with every precaution necessary to insure exactness, and at times when the fluctuations in the direction and intensity of the magnetic force were very small; and their results verify the conclusions above obtained, as applied to the case of small magnets.

Dr. Apjohn next read the following letter, which he had received from Captain Boileau, superintendent of the Magnetic Observatory at Simla, in India.

" *Simla, March*, 7, 1842.

" My dear Sir,

" I have the pleasure of forwarding to you, through the Government of India, a complete set of hygrometric tables,